Distance/Length

$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Midpoint

$M = \left(\underbrace{\frac{x_1+x_2}{2}}_{= x_m}, \underbrace{\frac{y_1+y_2}{2}}_{= y_m}\right)$

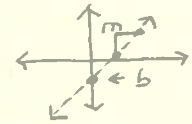
Finding Endpoint (visual method)

A (4,3) —— (7,-2) —— B ?

A (4, 3)
+3 −5
M (7, −2)
+3 −5
B (10, −7)

Equation of a Line

1) $y = mx + b$ (m = slope, b = y-int.)

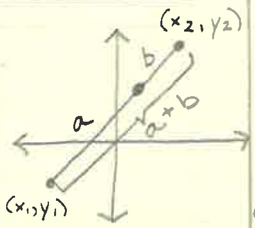


OR

2) $y - y_1 = m(x - x_1)$ — (x_1, y_1) = any point on line

Partitioning a Segment in a ratio of $a:b \rightarrow \frac{a}{a+b}$

$\left(x_1 + \frac{a}{a+b}\underbrace{(x_2 - x_1)}_{\Delta x},\ y_1 + \frac{a}{a+b}\underbrace{(y_2 - y_1)}_{\Delta y}\right)$



SLOPE (m)

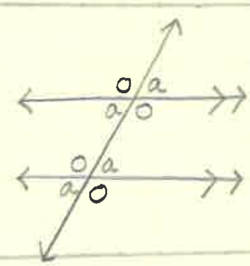
↗ m = ⊕ ↘ m = ⊖

↔ m = 0, ex: y = 3, y = −2

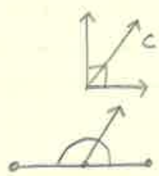
↕ m = undefined, ex: x = 4, x = −7

Calculating slope

$m = \frac{\text{rise}}{\text{run}}$ (w/ a graph) or $m = \frac{y_2 - y_1}{x_2 - x_1}$



a = a
o = o
a + o = 180



△ sum

$a + b + c = 180$

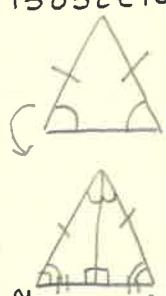
equilateral = equi∡ular

exterior ∡: $a + b = d$

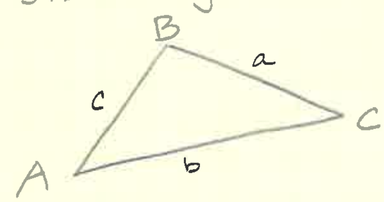
altitude drawn

Isosceles

≅ right △'s



Side-angle relationship



If $b > a > c$
then ∡B > ∡A > ∡C

Triangle Possible Side lengths

⊕ sum of any 2 sides > 3rd side

range of possible values: ex: 7, 4, x

⊕ sum (7 + 4) > x > ⊖ diff (7 − 4)

11 > x > 3

Congruent △'s (≅)

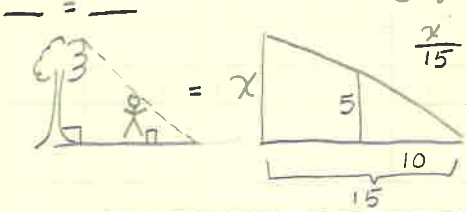
Proof methods: [ASA, SAS, SSS, HL, AAS] (HL: right △'s only)

once proven ≅ → "CPCTC" — all the corresponding parts of ≅ triangles are ≅ (as well)

Similar △'s (~) ⇒ proportions! matching!

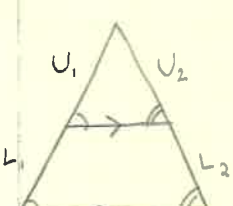
Proof methods: AA, (S)A(S), ASA (in proportion)

__ = __



$\frac{x}{15} = \frac{5}{10}$

$x = 7.5$



$\frac{U_1}{L_1} = \frac{U_2}{L_2}$

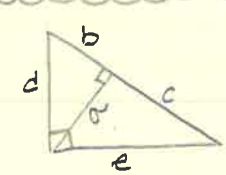
$\frac{U_1}{U_2} = \frac{L_1}{L_2}$

match it up!

△a ~ △b: $a \cdot \frac{b}{a} = b$ ($\frac{b}{a}$ = new/old)

ratio of sides = a:b in simplest form

Mean proportionals

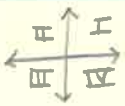


$\frac{\text{alt}}{\text{short}} = \frac{\text{alt}}{\text{long}}$

$\frac{b}{a} = \frac{a}{c}$

Leg rule: $\frac{\text{whole}}{\text{leg}} = \frac{\text{leg}}{\text{part}}$

$\frac{b+c}{d} = \frac{d}{b}$ or $\frac{b+c}{e} = \frac{e}{c}$



Transformations

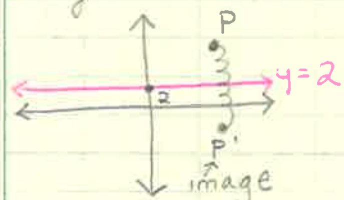
Translation = "shift"
$(x,y) \to (x+a, y+b)$ ↔ ↕

Reflection = "flip"

orientation changes
"lettering order"

$R_{x\text{-axis}}: (x,y) \to (x,-y)$
$R_{y\text{-axis}}: (x,y) \to (-x,y)$
$R_{y=x}: (x,y) \to (y,x)$
$R_{y=-x}: (x,y) \to (-y,-x)$
any other line: count!

ex:



Dilations: enlargement/reduction ⇒ creates similar shapes

$SF\ (k) = \frac{new}{OID}$

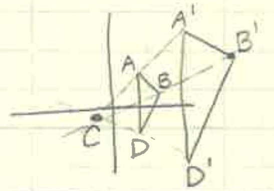
$\frac{CP'}{CP} = \frac{15}{5} = 3$

center of dilation (C)
pre-image (P)
image (P')
all collinear

dilation centered @ origin (0,0), w/ SF = k
$D_k: (x,y) \to (kx, ky)$

if not centered @ origin ⇒ graph to find center

ex:



$SF = \frac{new}{OID} = \frac{A'D'}{AD} = \frac{A'B'}{AB} = \frac{B'D'}{BD}$

Rotation = "turn"
around origin

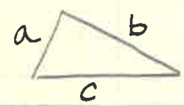
counter-clockwise / clockwise

$R_{90}\ (x,y) \to (-y,x) = R_{-270}$
$R_{180}\ (x,y) \to (-x,-y) = R_{-180}$
$R_{270}\ (x,y) \to (y,-x) = R_{-90}$

☆ or turn the paper to find new points

Converse of Pythagorean Theorem

$c^2 < a^2+b^2$	$c^2 = a^2+b^2$	$c^2 > a^2+b^2$
ACUTE	RIGHT	OBTUSE



Simplifying radicals

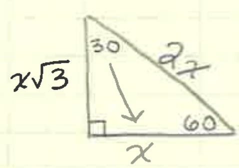
greatest perfect square

$\sqrt{20}$
$\sqrt{4}\ \sqrt{5}$
$2\sqrt{5}$

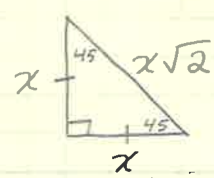
Perfect squares:
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
...

ex
$4\sqrt{5} \cdot 3\sqrt{10}$
$12\sqrt{50}$
$12\sqrt{25}\sqrt{2}$
$12 \cdot 5\sqrt{2}$
$60\sqrt{2}$

Special Right △'s

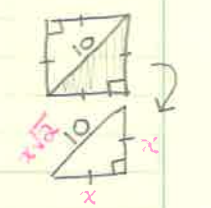


30-60-90



Isosceles

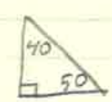
ex: square



$\sqrt{2} \cdot 10 = x\sqrt{2} \cdot \sqrt{2}$
$10\sqrt{2} = 2x$
$5\sqrt{2} = x$

Complementary ∠'s

$\sin a = \cos b$
$\sin b = \cos a$

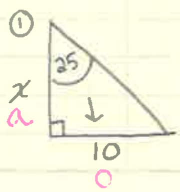


$\sin 40 = \cos 50$
$\sin 50 = \cos 40$

Trigonometry: SOH CAH TOA

If you're solving for the ∠ "x"
use inverse functions
$x = \sin^{-1}\left(\frac{o}{h}\right)$
$x = \cos^{-1}\left(\frac{a}{h}\right)$
$x = \tan^{-1}\left(\frac{o}{a}\right)$

ex: ①



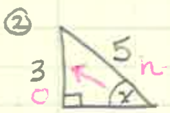
Toa → tan

$\tan 25 = \frac{10}{x}$

$x = \frac{10}{\tan 25}$

$x \approx 21.4$

②



Soh → sin

$\sin x = \frac{3}{5}$

$x = \sin^{-1}\left(\frac{3}{5}\right)$

$x \approx 36.87°$

Area Formulas (memorize)

Square: $A = s^2$

Rectangle: $A = l \cdot w$

Triangle: $A = \frac{1}{2}bh$

Circle: $A = \pi r^2$, $C = 2\pi r$

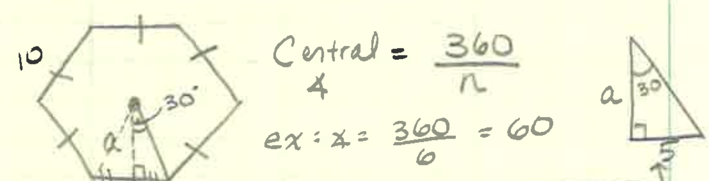
Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Rhombus; Kite: $A = \frac{1}{2}d_1 \cdot d_2$

any regular polygon over 4 sides

$A = \frac{1}{2}aP$

a = apothem (use trig to find a)
P = Perimeter



Central ∠ = $\frac{360}{n}$

ex: ∠ = $\frac{360}{6}$ = 60

Volume

B = area of the base

Prism/Cylinder: $V = Bh$

Pyramid/Cone: $V = \frac{1}{3}Bh$

Sphere: $V = \frac{4}{3}\pi r^3$

Lateral Area

area of all the faces or curved surface not including base

Cone: $LA = \pi r l$

Cylinder: $LA = 2\pi r h$

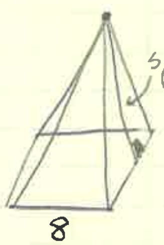
Surface Area

Break down 3D figure into net of shapes

$SA = LA + Base(s)$

Sphere: $SA = 4\pi r^2$ (great circle)

ex: Square Pyramid



LA = 4 Δ's

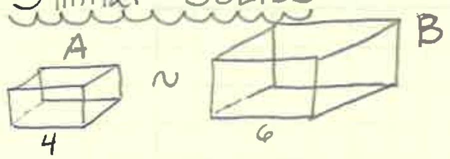
$A = \frac{1}{2}bh$

$\frac{1}{2}(8)(10)$

= 40 units²

× 4 → 160 units²

Similar Solids



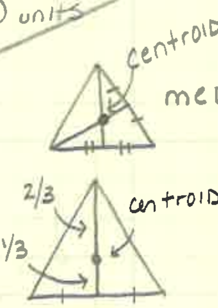
ratio of sides: SF: 4:6 → 2:3

Perimeter: SF = ratio of Perimeters $\frac{2}{3} = \frac{P_A}{P_B}$

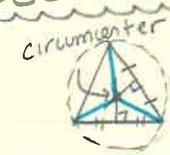
Area (SA or LA): SF^2 = ratio of areas $\frac{2^2}{3^2} = \frac{A_A}{A_B}$

Volume: SF^3 = ratio of volumes $\frac{2^3}{3^3} = \frac{V_A}{V_B}$

Special Segments

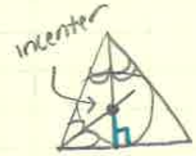


medians



⊥ Bisectors

circumcenter equidistant from vertices



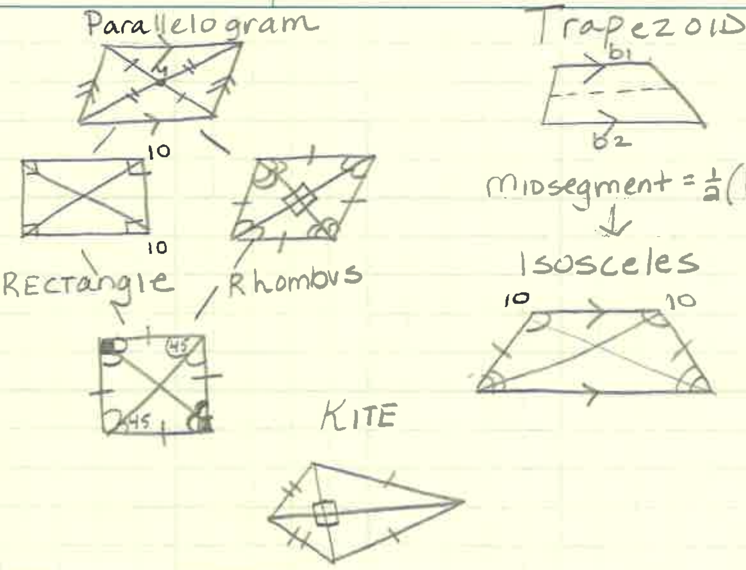
∠ Bisectors

incenter equidistant from sides

altitudes

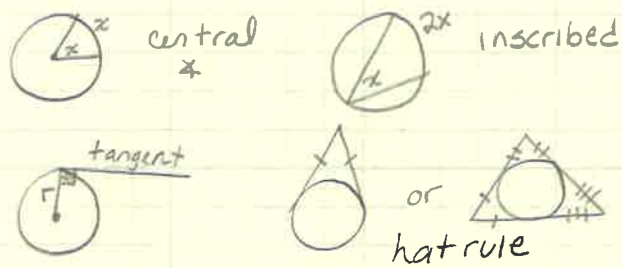
Polygons

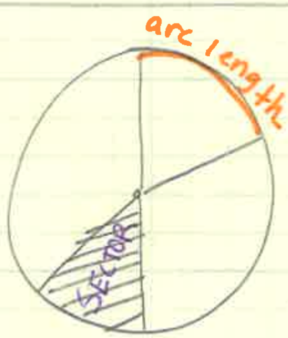
∡'s	Sum	EACH
Interior	$180(n-2)$	$\frac{180(n-2)}{n}$
Exterior	360	$\frac{360}{n}$



Circles

$(x-h)^2+(y-k)^2=r^2$

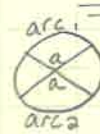




$\frac{\text{arc length}}{C\ (2\pi r)} = \frac{\text{central}\ ∡}{360}$

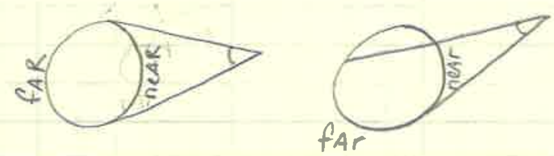
$\frac{\text{Area of sector}}{A_{\odot}\ (\pi r^2)} = \frac{\text{central}\ ∡}{360}$

∡ measures

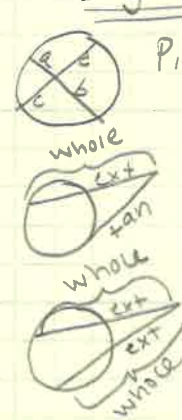


$a = \frac{1}{2}(\text{sum arcs})$

external ∡ $= \frac{1}{2}(\text{far} - \text{near})$



Segment Length



Piece × rest = Piece × rest
$a \cdot b = c \cdot d$

ext(whole) = tan²

ext(whole) = ext(whole)

Population Density = $\frac{\#\ \text{People}}{\text{Area of Land}}$

Degrees → Radians

$D \cdot \frac{2\pi}{360}$

Radians → Degrees

$R \cdot \frac{360}{2\pi}$